Instabilities in Circumstellar Discs

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Abstract.

I review recent results from the theory of angular momentum transport and explore their consequences for the evolution of circumstellar discs. In particular, numerical experiments show that the efficiency of angular momentum transport is sensitive to the presence of Ohmic diffusion. Since electron density, and therefore conductivity, increases sharply at 10^3 K due to ionization of the alkali metals, there is the possibility of unstable accretion in this temperature range. Illustrative models exhibit such instabilities and produce episodes of enhanced accretion similar to FU Orionis outbursts.

1. Introduction

Recent observational developments, including the discovery of extrasolar planets and the deployment of sensitive new near and mid infrared instruments, have heightened theoretical interest in circumstellar discs. The phenomenology of these discs is becoming increasingly complicated, and includes (moving outward from the young star) a magnetosphere, jet, centrifugally supported disc, disc wind, dust layer at the disc midplane, protoplanets and their attendant minidiscs, infall, and in many cases a companion star with its own disc, etc.

One of the main theoretical unknowns in this field is what drives the evolution of the disc. Observation gives only broad constraints in the form of timescales for disc dispersal (e.g. Strom 1994) and the spectral energy distribution (e.g. Adams et al. 1987), although observations are beginning to resolve portions of nearby discs (e.g. Rodriguez et al. 1998) In a centrifugally supported disc evolution is governed by angular momentum transport; without torques the disc gas would remain in orbit and not accrete, as is observed (Gullbring et al. 1998).

An equation governing disc evolution can be developed by combining the equations of angular momentum and mass conservations, assuming a thin disc ($H/r \ll 1$; $H \equiv$ scale height, $r \equiv$ local radius), and integrating vertically through the disc. The result is

$$\partial_t \Sigma(r,t) = \frac{2}{r}\partial_r\left(\frac{\Omega}{r\kappa^2}\partial_r(r^2 W_{r\phi}) - \frac{\Omega}{\kappa^2}\tau\right) + \dot{\Sigma}_{ext}, \quad (1)$$

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where $\Sigma \equiv$ surface density, $\Omega \equiv$ local rotation frequency, $\kappa \equiv$ epicyclic frequency. $W_{r\phi}, \tau$, and $\dot{\Sigma}_{ext}$ represent three physical effects that influence disc evolution. Notice that we have integrated out vertical evolution (e.g. dust settling).

The first term in equation (1) expresses the effect of $W_{r\phi}$, the turbulent shear stress or "anomalous viscosity," which causes angular momentum diffusion. It can be written

$$W_{r\phi} = \frac{1}{2\pi} \int d\phi \int dz \, \left(\rho v_r \delta v_\phi - \frac{1}{4\pi} B_r B_\phi + \frac{1}{4\pi G} \delta g_r g_\phi \right), \tag{2}$$

The first term in parentheses is the "Reynolds stress", which is the fluid's contribution to a.m. diffusion, the second term is the "Maxwell stress" arising from the magnetic field $\mathbf{B}$, and the final term is the gravitational stress arising from the gravitational field $\mathbf{g}$. The theoretical challenge here is, of course, to understand the physical processes that give rise to these stresses and to estimate their magnitude. There has been much progress on this front in the last few years, detailed below.

The shear stress is often characterized via a dimensionless $\alpha = W_{r\phi}/(\Sigma c_s^2)$, where c_s is a characteristic sound speed (N.B. a variety of definitions for α are in current use). It is likely that $\alpha < 1$, since $\alpha \gg 1$ would imply an energy density of order αP which couples to the matter in such a way as to transfer angular momentum but not increase the scale height. Put differently, it would imply that the off-diagonal components of the stress tensor are $\gg$ the diagonal components. Even $\alpha \approx 1$ would be somewhat surprising. If provided by a magnetic field oriented at an optimum $45°$ angle to the radius vector, it would imply a magnetic pressure of order the total pressure, a circumstance which is likely to be highly unstable to magnetic Rayleigh-Taylor instabilities (Newcomb 1961).

The second term in equation (1) expresses the effect of the external torque per unit area τ. This torque may arise in a variety of ways, but two which have received extensive attention are MHD winds (e.g. Blandford & Payne 1982, Pudritz & Norman 1983, Shu et al. 1994) and tidal torques (e.g. Lin & Papaloizou 1979, Goldreich & Tremaine 1980, others elsewhere in this volume). Since collimated winds are observed in young stellar objects, there is a strong case to be made for the importance of MHD winds. What is not clear is where these winds are launched. Optical jets, for example, have velocities characteristic of the gravitational potential close to the stellar surface. The relative importance of τ and $W_{r\phi}$ in disc evolution is now one of the most interesting topics in disc theory.

In recent years less progress has been made in understanding how MHD winds are launched than in understanding internal stresses because the wind problem is more difficult: it is global, yet intimately coupled to the internal dynamics of the disc. To solve it, one must understand not only the boundary conditions at the stellar surface and at large radius, but also the details of the transition region between disc and wind, how field is transported through a turbulent disc, and how that external field influences turbulence in the disc. The solution to this problem may be one of the first results to emerge from planned global numerical studies of magnetized discs, which are now just at the limits of computational feasibility.

The final term in equation (1) expresses the effect of direct additions and subtractions of mass from the surface of the disc. Mass loss in winds can contribute, as can infall from the parent molecular cloud. The latter depends on the initial conditions for star formation.

In comparison to other astrophysical discs (in CVs, X-ray binaries, and the inner parts of AGNs), circumstellar discs are quite cold. For reference, consider a "typical" solar nebula model (remember that conditions in the disc can change dramatically over short and long timescales– there is no set of physical conditions which describes the nebula over its entire lifespan). which has has surface density $\sim 10^3\,\mathrm{g\,cm^{-2}}$ and temperature $\sim$ 300 K at 1 AU. The density is then $\rho \approx \Sigma\Omega/(2c_s) \approx 10^{-9}\,\mathrm{g\,cm^{-3}} \approx 2\times 10^{14}\,\mathrm{cm^{-3}}$. An equipartition magnetic field would have $B = \sqrt{8\pi P} \approx 3\,\mathrm{G}$. Notice that at this density the grain-grain collision time for a population of $0.1\,\mu$m grains containing 0.01 of the gas mass in a laminar fluid is about 6 days, so the grain mass spectrum will almost certainly be evolved. This will be relevant later, when we consider the ionization state of the nebula.

2. MHD Turbulence

The last 7 years have seen significant advances in our understanding of the origin of the anomalous viscosity. These include: (1) the discovery that weakly magnetized discs are unstable (Balbus & Hawley 1991); (2) the demonstration, via numerical experiment, that this instability leads to turbulence and outward angular momentum transport (Hawley et al. 1995, 1996, Brandenburg et al. 1995); (3) the realization that convection does not transport angular momentum (Ryu & Goodman 1992, Stone & Balbus 1996, Cabot 1996); (4) numerical studies that show that Keplerian discs are probably not nonlinearly unstable, as was once thought (Balbus et al. 1996). These points are reviewed in detail by Balbus & Hawley (1998), by Gammie (1998a,b) and Brandenburg (1998).

While enormous strides have been made, it is still not possible to calculate $\alpha(T, \Sigma, \Omega)$. Indeed, it is not even clear that such a function exists (see the discussion of Gammie 1998b; also Armitage 1998). It might, for example, be necessary to write an expansion such as

$$\alpha = \alpha_0(T, \Sigma, \Omega) + c_1 \frac{d \ln T}{d \ln r} + c_2 \frac{d \ln \Sigma}{d \ln r} \ldots, \tag{3}$$

and so on. It is easy to invent expansions in terms of the various dimensionless disc parameters, including $\alpha \sim (H/R)^{1.5}$, which is popular in studies of cataclysmic variables.

Nevertheless, the numerical experiments done to date suggest (1) that $\alpha \gtrsim 0.01$ in a fully ionized fluid, and (2) that α depends on the local mean field (again, see Gammie 1998a). While future numerical experiments may provide us with an expression such as equation (3), another way to explore the consequences of the theory is to ask where MHD turbulence might fail, and where it might be dominated by other transport processes.

3. Ohmic Diffusion

In particular, at low ionization fraction linear theory shows that the Balbus-Hawley instability can be overwhelmed by Ohmic diffusion (see, e.g., Jin 1996, Gammie 1996) or ambipolar diffusion (Blaes & Balbus 1994). Which of these dissipative processes dominates depends on the disc density and ionization fraction and in particular on the value of the Hall parameter $\equiv (\omega_c \tau)_i$ for charged particle i, where ω_c is the cyclotron frequency and τ is the timescale for i to lose its momentum by collisions. If all the Hall parameters are $\ll 1$, then Ohmic diffusion dominates. Charged particles lose their momentum in collisions much more rapidly than they spiral around magnetic field lines. If all Hall parameters are $\gg 1$, then ambipolar diffusion dominates; in between lies the Hall regime studied in linear theory by Wardle (1998) and Wardle & Ng (1998). A nice discussion of these various possibilities is given in Goldreich & Reisenegger (1992) in the unlikely context of magnetic field decay on neutron stars. Ohmic diffusion probably dominates in the densest parts of circumstellar discs.

Ohmic diffusion kills the Balbus-Hawley instability when the growth rate at large wavelength, $\sim kV_A$ ($V_A = B/\sqrt{4\pi\rho} \equiv$ Alfven speed), matches the decay rate due to Ohmic diffusion, $\sim k^2\eta$. Here η is the resistive diffusivity, which appears in the induction equation as

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{v} \times \mathbf{B}) - \nabla \times (\eta \nabla \times \mathbf{B}). \tag{4}$$

It is convenient to use the magnetic Reynolds number

$$Re_M \equiv \frac{c_s H}{\eta} \tag{5}$$

to characterize the diffusivity, rather than η itself. Notice that various definitions of Re_M are used in the literature. Since the last wavelength to be stabilized by diffusion is the longest one available, $\sim H$, the instability will be shut off when

$$Re_M \lesssim \frac{c_s}{V_A}. \tag{6}$$

This discussion assumes that the strength of the magnetic field is a known parameter to be input on the right hand side of equation (6). But if the field strength is set by dynamo activity in the disc (rather than by currents outside the disc), then the problem becomes nonlinear: $V_A = V_A(Re_M)$. Numerical studies by Hawley et al. (1996) and by Fleming & Stone (1998) suggest that in the absence of externally generated field, MHD turbulence dies away for $Re_M \lesssim 10^4$ (although this result should be treated with caution, for reasons discussed in Gammie 1998a,b).

4. Ionization of Circumstellar Discs

So where in circumstellar discs might MHD turbulence be shut off? In H_2 gas, the resistivity is given by $\eta \approx 6.5 \times 10^3 x^{-1}\,\mathrm{cm}^2\,\mathrm{sec}^{-1}$, where $x = n_e/n_H$ is the electron fraction. Using the typical nebula conditions given earlier, we find that $Re_M > 1$ for $x \gtrsim 10^{-13}$! Where is this condition met?

It is easiest to begin by considering the situation where nonthermal ionization is unimportant. Then the ionization fraction can be estimated from the Saha equation (see the detailed calculations of Umebayashi [1983]). At low temperatures most free electrons are supplied by those alkali metals (K, Na, Ca) that have the lowest ionization potential and are most abundant. The resulting ionization fraction is weakly dependent on density, but exponentially sensitive to temperature. For typical densities Re_M exceeds unity near 10^3 K, and rises rapidly with temperature thereafter.

The inner 0.1 AU of discs around solar-mass stars are likely to be warmer than 10^3 K. At larger radius it becomes increasingly difficult for the disc to sustain such high temperatures if it is heated by internal turbulent dissipation alone. If the disc is in thermal equilibrium and the opacity $\kappa = 0.1 T^{1/2}\,\mathrm{cm}^2\,\mathrm{g}^{-1}$ (Bell & Lin 1994), then $T > 10^3$ K requires $\Sigma > 1100 r_{\mathrm{AU}}^{3/4} (\alpha/0.01)^{-1/2}\,\mathrm{g\,cm}^{-2}$. For $r \gtrsim 10$ AU the required disc mass exceeds the stellar mass.

The case where ionization is nonthermal is more difficult to treat. Ionization may result from cosmic rays (e.g. Gammie 1996), X-rays (Glassgold et al. 1997), or radioactive decay (Hayashi 1981, Stepinski 1992). Cosmic ray ionization is exponentially attenuated with a characteristic column $\simeq 100\,\mathrm{g\,cm}^{-2}$ (Umebayashi & Nakano 1981). This led me (Gammie 1996) to propose a model in which accretion proceeded only in surface layers with about this column. Significant uncertainties remain in the model, however. The recombination rate depends strongly on the abundance of small grains. If there are enough small grains, they will soak up all the free charge and give $Re_M < 1$ even in this surface layer. As noted above, however, the grain population will have evolved considerably from its interstellar state. It is also not clear whether the galactic cosmic ray flux can penetrate through the stellar or disc wind to the surface of the disc. Shielding of external cosmic rays may be compensated for by generation of energetic particles within the young star-disc system, but the flux of such particles is difficult to estimate.

X-ray ionization produces an ionized layer of depth similar to cosmic rays (Glassgold et al. 1997). Here it is unclear whether most young stars continuously produce enough hard X-rays; episodic ionization is probably insufficient to maintain MHD turbulence. Radioactive decay is probably too weak to give good ionization (again, see Hayashi 1981, Stepinski 1992, Gammie 1996), although the ionization fraction is sensitive to the abundance of ^{26}Al. It is also sensitive to the abundance of the dust which contains the important isotopes (^{40}K and ^{26}Al), so if there is a significant increase in the gas to dust ratio near the midplane due to dust settling, then there is a possibility of enhanced activity in the dust layer.

To summarize, hot ($T > 10^3$ K) parts of the disc are likely to be well-coupled to the magnetic field, while cold parts of the disc *might* be ionized in surface layers of 30—100 $\mathrm{g\,cm}^{-2}$. A large fraction of the mass of the disc may thus lie in a "dead zone" in which MHD turbulence is not present. Then $\alpha \approx 0$ in this region, unless some other transport process is active.

5. Gravitational and Hydrodynamic Instabilities

What non-MHD transport processes might be important in the cold, outer disc? Leaving out tidal interactions with gravitationally bound objects in the disc, two possibilities stand out: gravitational instability and global hydrodynamic instability.

Local gravitational instability sets in for a thin, fluid disc when Toomre's parameter

$$Q \equiv \frac{c_s \Omega}{\pi G \Sigma} < 1, \tag{7}$$

(there are order unity correction for finite thickness discs) or, scaling to parameters appropriate to discs,

$$\Sigma \gtrsim 5.6 \times 10^4 (M/M_\odot)^{1/2} (T/100\,\mathrm{K})^{1/2} r_{\mathrm{AU}}^{-3/2}\,\mathrm{g\,cm^{-2}}, \tag{8}$$

or, taking $M_{disc} \equiv \pi r^2 \Sigma$,

$$M_{disc} \gtrsim 0.02 (M/M_\odot)^{1/2} (T/100\,\mathrm{K})^{1/2} r_{\mathrm{AU}}^{1/2}\, M_\odot. \tag{9}$$

Q is also a good approximate indicator of global stability, even though locally stable discs are often globally unstable. Condition (9) is difficult to satisfy in circumstellar discs: it requires a cold, massive disc. These conditions may only be achieved in very young discs, if then. Evaluating (9) requires a knowledge of the surface density (set by initial conditions, disc evolution, and infall) and of the central temperature (set by turbulent heating and external illumination).

Let us suppose that gravitational instability *does* set in. What is the nonlinear outcome, and does it lead to transport and heating of the disc? In evaluating the nonlinear outcome, it is important to ask how the disc becomes unstable: by cooling (what controls the disc temperature?) or by mass loading.

This problem has long been of interest to students of galactic structure (e.g. Goldreich & Lynden-Bell 1965). Numerical studies of cooled, collisionless particle discs (e.g. Sellwood & Carlberg 1984, Toomre & Kalnajs 1991) indicate that gravitational instability sets in and becomes vigorous enough so that heating (increase of the particle velocity dispersion) balances imposed cooling, which for galaxies is often argued to be due to the introduction of young, low-velocity-dispersion stars. The heating is of course accompanied by angular momentum transport. The result is a disc which is marginally stable, with $Q \sim 1$ throughout. Fluid discs will not behave in exactly the same way; in particular, one must carefully consider what processes govern the gas temperature.

Most numerical studies of fluid discs have concentrated on nonlinear evolution of unstable modes in an adiabatic or isothermal disc. The isothermal approximation can be justified if the disc temperature is controlled by external heating, but in that case there is no reason for the disc to be marginally stable, as is usually assumed: the external heat flux that controls the disc temperature knows nothing of Q. It seems likely, then, that such externally heated discs become violently unstable if they become unstable at all. If they can regain stability, and not fragment, they are likely to do so by rearranging their surface density profile.

Many circumstellar discs may have their temperature controlled by internal heating. In this case, imagine the disc cooling towards $Q \sim 1$. Instability sets in, followed by shock heating of the gas. If the disc is to achieve a steady state, it must produce vigorous enough shock heating– and shock heating in the right place, at the low points in the potential– to balance radiative cooling. Following Alar Toomre's famous shearing-bits-and-patches experiments with particles, I have set up a number of numerical experiments to study thin, cooling, fluid discs. These experiments suggest that as long as the cooling time in the disc is short compared to the dynamical time (Ω^{-1}), the disc can recover from gravitational instability and produce enough shock heating to balance cooling. But if the cooling is too rapid, so that fluid clumps in the disc cool more rapidly than they can interact and shock, the disc fragments. These numerical results are consistent with the scenario set out by Shlosman & Begelman (1989) in the context of AGN discs.

Even if circumstellar discs are not so cool that they are gravitationally unstable, they can still be subject to global hydrodynamic instabilities such as the Papaloizou-Pringle (1984, 1985) and allied instabilities (see Narayan et al. 1986). Even if these instabilities are very weak, they could still change the nature of the "dead zone" by increasing α from 0 to, say, 10^{-5}.

6. Circumstellar Disc Evolution

What do these developments in the theory of angular momentum transport imply for the evolution of circumstellar discs? We can answer this, in part, using an illustrative evolutionary model for the disc.

The key difference between this model and earlier models is that we allow α to vary in a simple, physically motivated way. Thus

$$\alpha = \alpha_{\rm MHD} + \alpha_{grav}. \tag{10}$$

where $\alpha_{\rm MHD} = 10^{-2}$ if $T > 10^3\,{\rm K}$ or if the gas is located within $100\,{\rm g\,cm^{-2}}$ of the surface of the disc, and $\alpha_{grav} = e^{-Q^2}$. I will not defend this last expression too seriously; notice that gravitationally transport of angular momentum can be local only if $H/R \ll 1$, whereas $H/R \simeq 0.1$ in circumstellar discs. But the functional form of this gravitational α is not important (e^{-Q}, or Q^{-4}, works just as well); it simply acts to increase α so that Q never drops below 1.

The disc model can be evolved using a "two-zone" variation on equation (1) that neglects infall, mass loss, and external torques:

$$\partial_t \Sigma = \frac{1}{2\pi r}\partial_r(\dot{M}_a + \dot{M}_d). \tag{11}$$

Here a, d denote the "active" surface layer or the dead midplane region, $\dot{M}_{a,d} = 6\pi r^{1/2}\partial_r(\Sigma_{a,d}\nu_{a,d}r^{1/2})$ is the inward mass flux, and the turbulent viscosity $\nu_{a,d} = \alpha_{a,d}c_{s,a,d}H_{a,d}$ The active surface Σ_a is $200\,{\rm g\,cm^{-2}}$ if $\Sigma \equiv \Sigma_a + \Sigma_d > 200\,{\rm g\,cm^{-2}}$ and $\Sigma_d = 0$ otherwise. One also needs to account for vertical and radial transport of energy. I have used a radial turbulent mixing of entropy and vertical radiative diffusion based on analytic fits to the opacity by Bell & Lin (1994). The model is evolved using an explicit, finite difference numerical method.

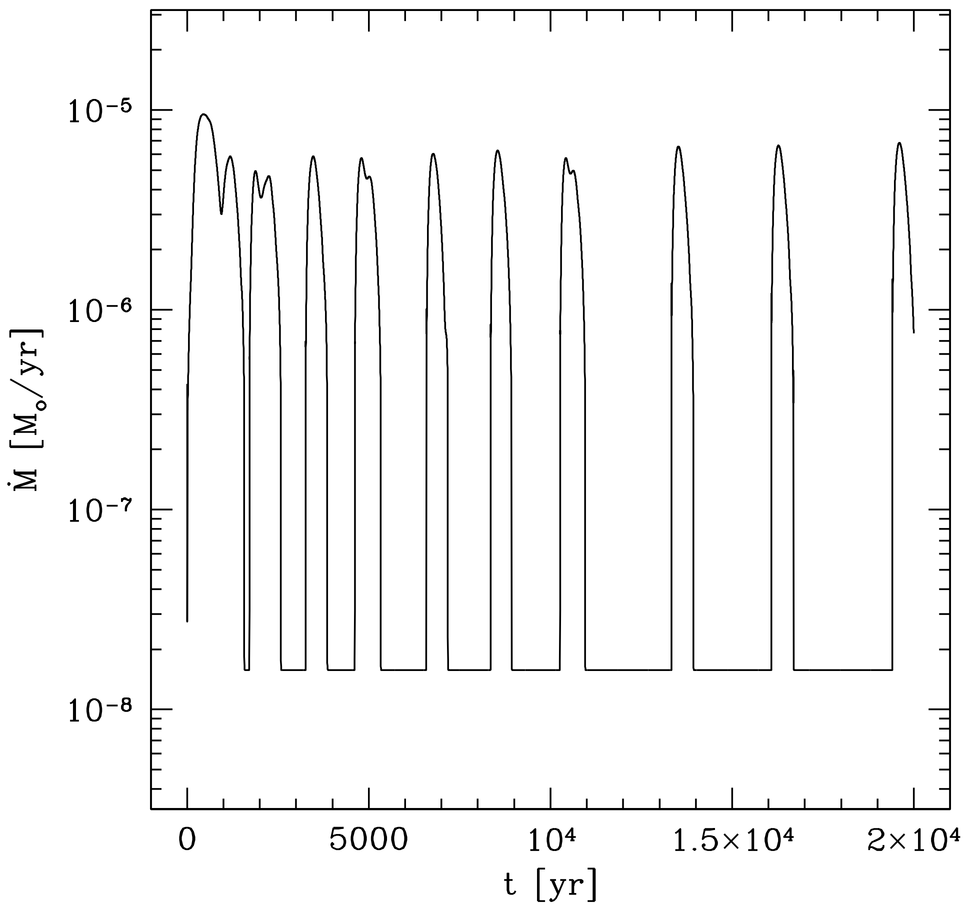

Figure 1. Variation of the accretion rate onto the central star (in solar masses/year) in a circumstellar disc model, as a function of time in years.

The accretion rate onto the central star in an example run is shown in Figure 1. The disc goes through episodes where it gradually fills in a region around $\sim 2\,\mathrm{AU}$ using cool material transported inward by weak gravitational instability. Eventually the midplane temperature in this region exceeds $10^3\,\mathrm{K}$, and it makes a transition to a hot state where the entire disc is MHD turbulent. Much of the disc inside this region then sloughs off onto the central star in a rapid accretion event that is reminiscent of FU Orionis outbursts. I will describe this model in more detail in a forthcoming paper.

The outcome is quantitatively but not qualitatively sensitive to the numerical resolution. It is sensitive to the input physics, however– introducing a relaxation timescale for $\alpha_{\rm MHD}$ (so that $\alpha_{\rm MHD}$ does not change instantaneously from 0 to 0.01), for example, produces a quite different lightcurve.

7. Conclusion

To sum up: (1) MHD turbulence can produce significant angular momentum transport in discs, but only if the ionization fraction is sufficiently high. (2) Many pieces of circumstellar discs are too poorly ionized– too cold and too dense– to couple to the magnetic field; these pieces will not be MHD turbulent. (3) Gravitational instability can also produce angular momentum transport, but *only* if the disc is sufficiently cold and massive (in this sense, gravitationally driven transport complements MHD turbulence). (4) The combination of these two processes in a simple, physically motivated model for disc evolution produces episodic accretion events that are similar to FU Orionis outbursts.

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