INSTRUCTIONS:
- Answer ALL questions.
- Read a problem carefully. Pause and think before you attempt to solve it.
- Always obtain an algebraic answer first; then and only then an arithmetic one. BUT: if asked for, numbers are just as important as algebraic answers; do not skip them!
- Specify your units. An arithmetic answer without units is a wrong answer.
- Do NOT rename symbols given to you in a problem.
- Explain briefly what you are doing; that way you may get credit even if your solution is wrong. If a grader cannot understand your reasoning, he/she may give you no credit.
- Write LEGIBLY; illegible answers receive NO credit.
- You may use the back of the exam pages for scratch work.

TABLE OF USEFUL CONSTANTS AND SOME FORMULAE:

- $M_{\text{Sun}} = 1.99 \times 10^{33} \text{ g}$
- $R_{\text{Sun}} = 6.96 \times 10^{10} \text{ cm}$
- $L_{\text{Sun}} = 3.83 \times 10^{33} \text{ erg s}^{-1}$
- $1 \text{ AU} = 1.50 \times 10^{13} \text{ cm}$
- $k_{\text{B}} = 1.38 \times 10^{-16} \text{ erg K}^{-1}$
- $m_p = 1.673 \times 10^{-24} \text{ g}$
- $m_n = 1.675 \times 10^{-24} \text{ g}$
- $e = 4.80 \times 10^{-10} \text{ esu} = 1.60 \times 10^{-19} \text{ C}$
- $\sigma_{\text{SB}} = 5.67 \times 10^{-5} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ K}^{-4}$
- $1 \text{ m} = 10^{2} \text{ cm} = 10^{6} \mu\text{m} = 10^{9} \text{ nm}$
- $M_{V(\text{Sun})} = +4.83 \text{ mag}$
- $T_{\text{Sun}} = 5778 \text{ K}$
- $R_{\text{Earth}} = 6.37 \times 10^{3} \text{ km}$
- $M_{\text{Earth}} = 5.97 \times 10^{27} \text{ g}$
- $M_{\text{Jupiter}} = 1.90 \times 10^{30} \text{ g}$
- $R_{\text{Jupiter}} = 7.14 \times 10^{4} \text{ km}$
- $H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$
- $\tau^2 = \frac{4\pi^2 a^3}{G(M_1 + M_2)}$
- $1 \text{ W} = 1 \text{ J s}^{-1} = 10^7 \text{ erg s}^{-1}$
1. **(25 points) Lagrangian Point**

The L₂ point lies beyond the Earth's orbit along the Earth-Sun line. At this point, the combined gravitational effect of the Earth and Sun balance the centrifugal force in a frame co-rotating with the Earth. Thus, a small body at the L₂ point remains stationary with respect to the Earth and Sun.

a) Write an expression for the centrifugal force in terms of the angular speed of the rotating frame \( \omega_{\text{rot}} \) and the distance from the Sun \( r \).

b) By balancing the centrifugal and gravitational forces, determine the distance of the L₂ point from the Earth (you may assume that this distance is much smaller than 1 AU and make suitable approximations).

c) Comment on why it might be advantageous to place a space telescope at the L₂ point.

2. **(25 points) Extrasolar Planet**

The star HD 27894 has an apparent V magnitude of 9.36 and an estimated mass \( M_A = 0.8 \, M_{\odot} \). Its radial velocity is observed to vary sinusoidally between −58 and +58 m s\(^{-1}\) with a period \( P = 18 \) days.

a) Assuming that a planet is causing these velocity variations, and that the planet's mass is much smaller than the star's, calculate the semimajor axis of the planet's orbit in AU.

b) Estimate the minimum mass \( M_B \) of the exoplanet. Give your answer in Jupiter masses. Why is this a minimum mass?

c) Suppose the planet is observed to transit in front of the star. Assuming it has the same density as Jupiter (1300 kg m\(^{-3}\)), calculate the fractional change in the star's flux (\( \Delta F/F \)) you would expect to measure during a transit. You may use the approximate relation \( R \propto M \) for low-mass stars.

3. **(25 points) Extinction and Reddening**

Consider an idealized, spherical cloud of atomic hydrogen with a number density equal to \( 10^3 \) cm\(^{-3}\) and a diameter equal to 2 pc.

a) Assume that the cloud is mixed with dust such that the ratio of dust mass to hydrogen mass is \( 3 \times 10^{-3} \). Calculate the number density of dust grains in the cloud, assuming that each dust particle has a radius \( a = 0.1 \mu m \) and a density of \( \rho_d = 3 \) g cm\(^{-3}\).

b) The “extinction efficiency” \( Q_\lambda \) is a dimensionless factor which multiplies the geometric cross-section \( \pi a^2 \) to yield the extinction cross-section. Assuming \( Q_\lambda = (\lambda/\lambda_V)^{-1.5} \), calculate the optical depth through the center of the cloud in the \( V \) and \( B \) bands. You may assume that \( \lambda_V = 0.55 \) μm and \( \lambda_B = 0.44 \) μm.

c) The extinction in magnitudes is given by \( A_\lambda = 1.086 \tau_\lambda \), where \( \tau_\lambda \) is the optical depth at wavelength \( \lambda \). Calculate \( A_V, A_B \), and the reddening \( E(B - V) = A_B - A_V \) through the center of the cloud.

d) Standard ISM textbooks give a “gas-to-dust” ratio \( N_H/A_V = 2 \times 10^{21} \) cm\(^{-2}\) mag\(^{-1}\) and an extinction-to-reddening ratio of \( A_V/E(B - V) = 3.1 \). Compare these “standard” ratios to the ratios you obtain for this cloud.
4. **(25 points) Jeans Mass**

The virial theorem \((2K + W = 0)\) can be used to estimate the maximum mass of a spherical cloud that is supported only by thermal pressure. Here \(K\) is the random (internal) energy of the system and \(W\) is the gravitational potential energy.

a) Write down an expression for the internal (or thermal) energy \(K\) of an atomic hydrogen cloud of total mass \(M\) and temperature \(T\). (Hint: It should involve only \(M\), \(T\), and physical constants.)

b) The gravitational potential energy for a sphere of constant density \(\rho\) can be derived by integrating in spherical shells outward from the center:

\[
W = -\int_{0}^{M} \frac{Gm(r)dm}{r},
\]

where \(m(r)\) is the mass within radius \(r\) and \(dm\) is the mass of a thin spherical shell at a distance \(r\) from the center of the sphere. Determine \(W\) in terms of \(M\) and \(\rho\).

c) Using the virial theorem, determine the maximum stable mass (known as the Jeans mass) for a spherical cloud of constant density and temperature. Express your answer in terms of \(\rho\), \(T\), and physical constants.

d) For a hydrogen cloud of number density \(n = 10^2\) cm\(^{-3}\), calculate the Jeans mass for \(T = 10^4\) K and \(T = 10^2\) K. Based on your results, what is likely to happen if a gas cloud cools at nearly constant density?

e) For an external confining pressure \(P_0\), the virial theorem can be written as \(2K + W - 3P_0V = 0\); where \(V\) is the volume of the cloud. Derive an expression for the confining pressure in terms of \(M\), \(T\), and the cloud radius \(R\). Is this a maximum or minimum external pressure for stability? Explain.