INSTRUCTIONS:
- Answer ALL questions.
- Read a problem carefully. Pause and think before you attempt to solve it.
- Always obtain an algebraic answer first; then and only then an arithmetic one. BUT: if asked for, numbers are just as important as algebraic answers; do not skip them!
- Specify your units. An arithmetic answer without units is a wrong answer.
- Do NOT rename symbols given to you in a problem.
- Explain briefly what you are doing; that way you may get credit even if your solution is wrong. If a grader cannot understand your reasoning, he/she may give you no credit.
- Write LEGIBLY; illegible answers receive NO credit.
- You may use the back of the exam pages for scratch work.

TABLE OF USEFUL CONSTANTS AND SOME FORMULAE:

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\begin{align*}
M_{\text{Sun}} &= 1.99 \times 10^{33} \text{ g} & G &= 6.67 \times 10^{-8} \text{ dyn cm}^2 \text{ g}^{-2} \\
R_{\text{Sun}} &= 6.96 \times 10^{10} \text{ cm} & 1 \text{ pc} &= 3.09 \times 10^{18} \text{ cm} \\
L_{\text{Sun}} &= 3.83 \times 10^{33} \text{ erg s}^{-1} & 1 \text{ yr} &= 3.16 \times 10^7 \text{ s} \\
1 \text{ AU} &= 1.50 \times 10^{13} \text{ cm} & c &= 3.00 \times 10^{10} \text{ cm s}^{-1} \\
k_B &= 1.38 \times 10^{-16} \text{ erg K}^{-1} & h &= 6.63 \times 10^{-27} \text{ erg s} \\
m_p &= 1.673 \times 10^{-24} \text{ g} = 938.3 \text{ MeV/c}^2 & m_e &= 9.11 \times 10^{-28} \text{ g} = 0.511 \text{ MeV/c}^2 \\
m_n &= 1.675 \times 10^{-24} \text{ g} = 939.6 \text{ MeV/c}^2 & m_\pi &= 139.6 \text{ MeV/c}^2 \\
e &= 4.80 \times 10^{-10} \text{ esu} = 1.60 \times 10^{-19} \text{ C} & 1 \text{ eV} &= 1.60 \times 10^{-12} \text{ erg} \\
\sigma_{SB} &= 5.67 \times 10^{-5} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ K}^{-4} & 1 \text{ J} &= 9.87 \times 10^3 \text{ L atm} = 10^7 \text{ erg} \\
1 \text{ m} &= 10^2 \text{ cm} = 10^6 \mu\text{m} = 10^9 \text{ nm} & 1 \text{ Å} &= 10^8 \text{ cm} = 10^{10} \text{ m} = 0.1 \text{ nm} \\
M_V(\text{Sun}) &= +4.83 \text{ mag} & M_B(\text{Sun}) &= +5.50 \text{ mag} \\
T_{\text{Sun}} &= 5778 \text{ K} & m_V(\text{Sun}) &= -26.5 \text{ mag} \\
R_{\text{Earth}} &= 6.37 \times 10^3 \text{ km} & R_{\text{Moon}} &= 1.74 \times 10^3 \text{ km} \\
M_{\text{Earth}} &= 5.97 \times 10^{27} \text{ g} & r_{\text{Earth-Moon}} &= 3.84 \times 10^5 \text{ km} \\
M_{\text{Jupiter}} &= 1.90 \times 10^{30} \text{ g} & M_{\text{Venus}} &= 4.90 \times 10^{27} \text{ g} \\
R_{\text{Jupiter}} &= 7.14 \times 10^4 \text{ km} & R_{\text{Venus}} &= 6.05 \times 10^3 \text{ km} \\
H_0 &= 100h \text{ km s}^{-1} \text{ Mpc}^{-1} & T &= (0.290 \text{ cm})/\lambda_{\text{max}} \text{ K} \\
\tau^2 &= \frac{4\pi^2 a^3}{GM_1 + M_2} & 1 \text{ W} &= 1 \text{ J s}^{-1} = 10^7 \text{ erg s}^{-1}
\end{align*}
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1. (25 points) Tidal Forces

The maximum orbital radius for a satellite is given by the Hill radius, for which the difference between the planet’s acceleration due to the Sun and the satellite’s acceleration due to the Sun equals the satellite’s acceleration due to the planet. (You may use two-significant figure accuracy in calculations in this problem.)

a) Calculate the Hill radius \( r_H \) for the Earth in AU, assuming that the Sun is much farther away than the distance of any satellite from the Earth.

b) Show that a telescope placed in orbit around the Sun at a distance of \( 1 r_H \) beyond the Earth’s orbit (along the Sun-Earth line) would experience the same acceleration as the Earth does. (This location is referred to as the \( L_2 \) Lagrangian point.)

c) Without doing a full calculation, discuss how the Hill radius for Venus should compare with the Hill radius for the Earth. Explain your result physically.

2. (25 points) Exoplanet Properties

A star with mass equal to the Sun’s is observed to exhibit a perfectly sinusoidal variation in its radial velocity, with a period of 44 days and an amplitude of 20 m s\(^{-1}\) (see figure below).

![Graph showing radial velocity over time](image)

a) Explain why this kind of variation of the radial velocity is indicative of a planetary companion. What is the implied orbital period of the planet?

b) Derive the minimum mass of the exoplanet that causes these variations in radial velocity. You may assume that the planet’s mass is much smaller than that of the star. Express your answer in units of the Jupiter mass.

c) For what range of orbital inclinations (with 90° being edge-on) would a transit be observable, in which the planet’s disk blocks (occults) a portion of the star’s disk? You may assume that the radius of the star is equal to that of the Sun and that the density of the planet is equal to that of Jupiter (i.e., 1.3 g cm\(^{3}\)).
3. (25 points) HI Emission

A neutral-hydrogen atom in the excited hyperfine state must wait an average of \(10^7\) yr before spontaneously emitting a 21-cm photon.

a) For a pure hydrogen cloud with number density \(10\ \text{cm}^{-3}\) and diameter 10 pc, estimate the total number of hydrogen atoms in the cloud.

b) If 75% of the atoms are maintained in the upper state, what is the luminosity of the cloud in 21-cm radiation (expressed in solar luminosities)?

c) Assuming that the 21-cm line is optically thin, calculate the ratio of the mass of the cloud (in solar masses) and the 21-cm luminosity (in solar luminosities). This can be thought of as the mass-to-light ratio in the 21-cm line.

4. (25 points) Ionization Equilibrium

HII regions surround hot, O-type (and B-type) stars that emit ionizing photons. In steady state, the total recombination rate within an HII region equals the ionization rate.

a) Estimate the time it takes a neutral atom within the HII region to become ionized. To do this, assume that the target atom has a cross section equal to \(10^{-17}\ \text{cm}^2\), that it is located at a distance of 5 pc from the star, and that the star emits \(10^{48}\) ionizing photons per second that can reach this distance.

b) Estimate the time it takes an electron within the HII region to recombine with a proton. Assume a proton number density of 100 \(\text{cm}^{-3}\), a recombination cross section of \(10^{-20}\ \text{cm}^2\), and a typical electron speed of 500 \(\text{km s}^{-1}\).

c) Equating the ionization and recombination rates, estimate the ratio of ionized to neutral hydrogen atoms within the HII region.